3. Complete Contracts II: Static Multilateral Contracting

Huihua NIE
School of Economics
Renmin University of China
www.niehuihua.com
niehuihua(at)vip.163.com

3.1 Moral Hazard III: Multiple agents

When consider contracting situations where a principle interacts with multi-agents, we actually consider not only optimal incentive provision, but also *organization design* and the theory of the firm, including supervision, cooperation, collusion, promotion, and etc. Of course, the problem becomes more complex because of strategic actions among agents. Recall the fable of "three monks without water" and think about how to prove it.

Following Holmstrom (1982), we shall consider in turn two diametrically opposite cases. The first case is pure moral hazard in a team where the output produced by the agents is a single aggregate output Q with conditional distribution $F(Q|a)$. The second case is that each agent produces an individual random output q_i which may be imperfectly correlated with the other agents' outputs.

3.1.1 Unobservable individual outputs: free-ride problem

Following Alchian and Demsetz (1972)'s basic idea that a firm is a team of production with hidden actions, we analyze the free-ride problem in team in Holmstrom (1982)'s formation.

Consider a risk neutral principal engaged in a contractual relation with n risk neutral agents, which produce some deterministic aggregate output $Q = Q(a_1, a_2, ..., a_n)$ by a vector of individual hidden efforts $a = (a_1, a_2, ..., a_n)$. Suppose that $\frac{\partial Q}{\partial a_i} > 0$, $\frac{\partial^2 Q}{\partial a_i^2} < 0$, $\frac{\partial^2 Q}{\partial a_i \partial a_j} > 0$ (it means that different agents' efforts are complementary). Agent's utility function is $U = w_i - c_i(a_i)$. Define a partnership to be a vector of output-contingent compensations for each agent, $w(Q) = (w_1(Q), w_2(Q), ..., w_n(Q))$ such that $\sum w_i(Q) = Q$.

The first-best vector of effort is uniquely defined by

$$a_i^* \in \arg\max Q(a^*) - \sum c_i(a_i^*)$$

$$\Rightarrow \frac{\partial Q(a^*)}{\partial a_i} = c_i'(a_i^*) \quad \text{for each } i \tag{8-1}$$

However, in Nash equilibrium under asymmetric information, each agent's optimal effort is defined by

$$a_i^{NE} \in \arg\max w_i(Q) - c_i(a_i^{NE})$$

$$\Rightarrow \frac{dw_i(Q)}{dQ}\frac{\partial Q}{\partial a_i} = c_i'(a_i^{NE}) \quad (8\text{-}2)$$

The expression (8-2) indicates that only if each agent gains full marginal return from his (optimal) effort a_i^*, the efforts level with Nash equilibrium will attain the first-best efforts level. Formally it requires that

$$\frac{dw_i(Q)}{dQ} = 1 \Rightarrow w_i(Q) = Q$$

But it contradicts the budget constraint condition that $\sum w_i(Q) = Q$, so the first best can't reach and $a_i^{NE} < a_i^*$. This is the truth of "three monks without water".

Budget breaker—Holmstrom contract

If we introduces a budget breaker/principal into the organization, who pay each agent $w_i(Q) = Q$, there will exist a Nash equilibrium where all agents supply their fist-best efforts and the third party—residual claimant—pays out $nQ(a^*)$ in equilibrium. It requires that each agent make an up-front payment z_i to the budget breaker such that

(PIR) $\sum z_i + Q(a^*) \geq n(Q^*)$

(AIR) $z_i \leq Q(a^*) - c_i(a_i^*)$

Hence we can rewrite AIR as

$$Q(a^*) - \sum c_i(a_i^*) \geq \sum z_i - (n-1)Q(a^*) \geq 0$$

It means that the first best can attain, so the Holmstrom contract is a profitable contract.

Comments

Holmstrom's observation that a team requires a budget breaker has been very influential which has been seen as a fundamental reason why a firm needs a residual claimant and why needs to seek outsider financing and why the principal doesn't make efforts in classical agency model. But Holmstrom's breaker is very different from Alchian-Demsetz's residual claimant, who participates in production activities in a classical capitalism firm.

More importantly, the Holmstrom contract is not real in reality, because of the following reasons: (1) The principal would get nothing if budget constraint balances or get negative payment if the firm or team performs better than best. **Actually the principal must pay the agents two parts—full**

marginal contribution to the agent who made the effort and the rest to other agents in order to keep budget balance. (2) It requires that agents have adequate wealth. (3) As Steven Cheung（张五常）pointed, sometimes the supervisor is not the boss, but hired by the workers. This is true among Sichuan boatmen.

Mirrlees contract

The contract is

$$\begin{Bmatrix} b_i, if \quad Q = Q(a^*) \\ k, otherwise \end{Bmatrix} \text{ and } k \geq c_i(a_i^*) - b_i \text{ for all } i.$$

There indeed exists a Nash equilibrium where all agents supply their first-best efforts under this contract, which assure $k = 0$ and $\sum b_i \geq \sum c_i(a_i^*)$ (Pareto condition). And if $Q(a^*) \geq \sum b_i$, the budget breaker has incentive to implement the contract. Interestingly, the Mirrlees contract is equivalent to a debt contract, where $D = Q(a^*) - \sum b_i$.

Comments

Although the Mirrlees contract is more realistic than the Holmstrom contract, and needn't huge wealth for agents, it is vulnerable to an important weakness—multiple equilibria. For example, if some agents shirk, other agents have to burden all the work to avoid penalties. Extremely, if all other agents do nothing, the agent i has to do all the work in order to provide $Q(0,...,\hat{a}_i,...,0) = Q(a^*)$. But the Holmstrom contract has no the problem.

Think

All the above contracts are realistic in China?

In reality, how can we solve the free-ride problem? Reputation? Informal organization?

We have the famous fable of "three monks", but why we don't has team production theory?

Reference

My contribution related to the free-ride problem is: 李金波、聂辉华和沈吉，2010，《团队生产、集体声誉和分享规则》，《经济学（季刊）》，第 9 卷第 3 期.

3.1.2 Observable individual outputs: relative performance evaluation

When the situation where only aggregate output is observable moves to the situation individual agent outputs are observable, the core issue of eliciting cooperation moves to controlling competition among agents. Under this kind of information structure, relative performance evaluation is used widely as an incentive scheme in all kinds of organizations or activities, such as school, company, government, sports. *Rank-order tournament* is a very common relative performance evaluation scheme.

Following Holmstrom (1979, 1982), consider the multiagent situation with two identical agents, each producing an individual output q_i by supplying effort a_i, where

$$x_1 = a_1 + \varepsilon_1 + \gamma\varepsilon_2$$

$x_2 = a_2 + \varepsilon_2 + \gamma\varepsilon_1$

Where, $\varepsilon \sim (0, \sigma^2)$. γ indicates common shock or luck. When $\gamma = 0$, we come back to the single-agent problem. Principal is risk neutral and both agents are risk averse and have CARA risk preferences represented by $u(w,a) = -\exp[-r(w - c(a)]$, where $c(a) = \frac{1}{2}ba^2$. Finally, we restrict attention to linear incentive schemes of the form

$w_1 = \alpha_1 + \beta_1 x_1 + \mu_1 x_2$

$w_2 = \alpha_2 + \beta_2 x_2 + \mu_2 x_1$

Notice that when $\mu = 0$, it is not relative performance evaluation. Given the symmetry of the principal's problem, we character the problem as follows:

$$\underset{a_1,\alpha_1,\beta_1,\mu_1}{Max}\ E(x_1 - w_1)$$

s.t. (IR) $E(-e^{-r(w-c(a_1))}) \geq u(\underline{w})$

(IC) $a_1 \in \arg\max_{a_1} E(-e^{-r(w-c(a_1))})$

To solve the program, firstly we compute the agent's certainty equivalent, that is

$$ACE = E[w_1 - c(a_1)] - \frac{1}{2}rVar(w_1)$$

$$= \alpha_1 + \beta_1 a_1 + \mu_1 a_2 - \frac{1}{2}ba_1^2 - \frac{1}{2}rVar(w_1)$$

$$Var(w_1) = Var[\beta_1(\varepsilon_1 + \gamma\varepsilon_2) + \mu_1(\varepsilon_2 + \gamma\varepsilon_1)]$$
$$= Var[(\beta_1 + \gamma\mu_1)\varepsilon_1 + (\gamma\beta_1 + \mu_1)\varepsilon_2]$$
$$= [(\beta_1 + \gamma\mu_1)^2 + (\gamma\beta_1 + \mu_1)^2]\sigma^2$$

$$\therefore ACE = \alpha_1 + \beta_1 a_1 + \mu_1 a_2 - \frac{1}{2}ba_1^2 - \frac{1}{2}r\sigma^2[(\beta_1 + \gamma\mu_1)^2 + (\mu_1 + \gamma\beta_1)^2]$$

Maximize ACE with respect to a_1, and in the Nash equilibrium, we have

$$a_1 = \frac{\beta_1}{b} \text{ and } a_2 = \frac{\beta_2}{b}.$$

Substituting for a_i in ACEs, we obtain each agent's equilibrium payoff, and for agent 1 it is

$$\alpha_1+\frac{1}{2}\frac{\beta_1^2}{b}+\frac{\mu_1\beta_2}{b}-\frac{1}{2}r\sigma^2[(\beta_1+\gamma\mu_1)^2+(\mu_1+\gamma\beta_1)^2]$$

Then the principal's problem is maximize $\{\frac{\beta_1}{b}-(\alpha_1+\frac{\beta_1^2}{b}+\frac{\mu_1\beta_2}{b})\}$. Let $\underline{w}=0$ and substitute for α_1, we have

$$\underset{\beta_1,\mu_1}{Max}\{\frac{\beta_1}{b}-\frac{1}{2}\frac{\beta_1^2}{b}-\frac{1}{2}r\sigma^2[(\beta_1+\gamma\mu_1)^2+(\mu_1+\gamma\beta_1)^2]\}$$

We solve the problem sequentially:

(1) For any given β_1, μ_1 is determined to minimize risk;

(2) Optimize object function and get optimal β_1.

At first, differentiating $\frac{1}{2}r\sigma^2[(\beta_1+\gamma\mu_1)^2+(\mu_1+\gamma\beta_1)^2]$ with respect to μ_1, we have

$$\mu_1=-(\frac{2\gamma}{1+\gamma^2})\beta_1$$

It implies that if $\gamma>0$ (complementary tasks, e.g., umbrella and boots during the rainy season in southern China), $\mu_1<0$ (negative incentive); $\gamma<0$ (substituting tasks, e.g., air conditioner vs. boots), $\mu_1>0$ (positive incentive). That is to say, if two agents' outputs are positively correlated, the optimal μ_1 is negative; verse vice. The optimal incentive scheme reduces agent 1's exposure to a *common shock*. For an organization, introducing a form of relative performance evaluation is to reduce each agent's risk exposure by *filtering out* the common shock. Of course, the presumption that relative performance evaluation works is individual outputs are not independent ($\gamma\neq 0$).

Secondly, we substitute for μ_1 in the object function and solve

$$\underset{\beta_1}{Max}\{\frac{\beta_1}{b}-\frac{1}{2}\frac{\beta_1^2}{b}-\frac{1}{2}r\sigma^2\beta_1^2\frac{(1-\gamma^2)^2}{1+\gamma^2}\}$$

We have

$$\beta_1=\frac{1+\gamma^2}{1+\gamma^2+rb\sigma^2(1-\gamma^2)^2}$$

Notice that when $\gamma = 0$, it reduces to the classical situation that $\beta_1 = \frac{1}{1 + rb\sigma^2}$; when $\gamma = 1$ (or -1), $\beta_1 = 1$. In other words, when each agent's outputs are almost entirely affected by the common shock, by filtering out this common shock, the optimal incentive scheme can then almost eliminate each agent's exposure to risk and thus approximate first-best incentives by letting $\beta_1 = 1$.

Notice also that the main reason for relative performance evaluation is not to induce competition but induce information and low the risk exposure.

3.1.3 Tournaments

Perhaps the most prevalent form of relative performance evaluation in reality is tournament, which base compensation only on an ordinal ranking of individual agents' outputs. One obvious advantage of tournament schemes if that ordinal ranks are easy to measure and hard to manipulate for agents or the principal.

Risk neutrality and no common shock

We follow Lazear and Rosen (1981), and suppose that two agents are risk neutral and individual outputs are independently distributed (NO common shock). It seems that tournaments should be inefficient, but Lazear and Rosen show that the first-best outcome can be implemented using a tournament.

Let $x_i = a_i + \varepsilon_i$, where ε_i is i.i.d. with mean 0 and variance σ^2. So, the first-best effort level for each agent is given by the FOC $\max[a_i - c(a_i)] \Rightarrow c'(a_i) = 1$.

We can compare two methods of payment: purely individual performance-related pay (say, *piece rates*) and relative-performance-based pay, *tournaments*.

Obviously, when pay is set as $w_i = \alpha + x_i$, the first method attains the first best, because the agent is risk neutral and becomes the residual claimant.

The tournament is structured as follows: the agent with higher output gets a fixed wage α plus a prize W, while the agent with lower output gets only α. So agent i's expected payoff is

$w_i = \alpha + pW - c(a_i)$, where $p = \Pr(x_i > x_j) = \Pr(a_i - a_j > \varepsilon_j - \varepsilon_i) = H(a_i - a_j)$

H is the cumulative distribution of $(\varepsilon_j - \varepsilon_i)$, which has mean 0 and variance $2\sigma^2$. An agent's best response is

$$W\frac{dp}{da_i} = c'(a_i) \quad \text{or} \quad Wh(a_i - a_j) = c'(a_i)$$

In Nash equilibrium, $a_i^* = a_j^*$, so in order to attain first best, we must specify a prize

$$W = \frac{1}{h(\square)}$$

Comments

The mechanism by which the first best is achieved looks very different in the two incentive schemes however: under piece rates, it looks like the agent has direct control over her compensation, up to a random shock: she is directly controlling the mean of her compensation. Instead, in a symmetric tournament, both agents choose the same effort level and it is solely chance that determines who the winner is!

Risk-averse agents and common shock

We have shown that when agents are risk neutral and when their outputs are independent, piece rates and tournaments are equally efficient incentive schemes. However, Green and Stokey (1983) show that this equivalence breaks down when agents are risk averse. They establish that tournaments are then dominated by piece-rate schemes when agents' outputs are independent, but tournaments may dominate piece rates when a common shock affects agents' performance. Tournaments are then also approximately second-best optimal incentive schemes when the number of agents is large. The reason why a tournament's efficiency improves as the number of agents in play increases is that the sophistication of the tournament increases with the number of agents. We shall compare the two incentive schemes by evaluating the cost to the principal of implementing a given effort for both agents.

Suppose that $x_i \in \{0,1\}$, and the probability of success is $\Pr(x_i = 1) = \xi a_i$ and $\Pr(x_i = 0) = \xi(1 - a_i)$ for failure. When common shock appears, all agents' output is 0 with probability $1 - \xi$. Two identical risk-averse agents' utility function are $u(w) - c(a)$. A feasible contract is $w = \{w_{00}, w_{10}, w_{01}, w_{11}, w_c\}$, where when output was affected by a common shock w_c is paid, otherwise w_{ij} is paid.

So, agent's incentive constraint is

$$a_1 \in \arg\max_a \{\xi[a_1(1-a_2)u(w_{10}) + a_1 a_2 u(w_{11}) + (1-a_1)(1-a_2)u(w_{00})$$
$$+(1-a_1)a_2 u(w_{01})] + (1-\xi)u(w_c) - \frac{1}{2}ba_1^2\}$$

FOC:

$$\xi[(1-a_2)u(w_{10}) + a_2 u(w_{11}) - (1-a_2)u(w_{00}) - a_2 u(w_{01})] = ba_1$$

The principal's problem reduces to

$$\underset{\{w_{00}, w_{10}, w_{01}, w_{11}, w_c\}}{Min} \{\xi[a_1(1-a_2)u(w_{10}) + a_1 a_2 u(w_{11}) + (1-a_1)(1-a_2)u(w_{00})$$
$$+(1-a_1)a_2 u(w_{01})] + (1-\xi)u(w_c)\}$$

s.t.

$$\xi[a_1(1-a_2)u(w_{10})+a_1a_2u(w_{11})+(1-a_1)(1-a_2)u(w_{00}) \\ +(1-a_1)a_2u(w_{01})]+(1-\xi)u(w_c)-\frac{1}{2}ba_1^2\geq 0 \qquad \text{(IR)}$$

$$\xi[(1-a_2)u(w_{10})+a_2u(w_{11})-(1-a_2)u(w_{00})-a_2u(w_{01})]=ba_1 \qquad \text{(IC)}$$

Solve the program, when $\xi=1$ (no common shock), we get

$w_{10}=w_{11}$ and $w_{00}=w_{01}$

Those mean that agents get full assurance which like as under symmetric information structure.

When $0<\xi<1$, we get

$w_{00}\neq w_c$

We are thus able to draw the following observations:

(1) When there is no common shock, then any relative-performance-evaluation scheme, which would result in either $w_{10}\neq w_{11}$ or $w_{00}\neq w_{01}$ or both, is *suboptimal*. Only a piece-rate scheme is optimal. Because relative-performance-evaluation scheme only increase the agent's risk exposure without improving his incentives.

(2) If a general relative-performance-evaluation scheme is suboptimal, then a fortiori a tournament is suboptimal. Indeed, a tournament would specify that $w_{11}=w_{00}=w_c=T$ (tie), $w_{10}=W$ (winner), and $w_{01}=L$ (loser). Unless $L=T=W$, it would not satisfy the preceding optimality conditions, however agents would have no incentive to work.

(3) In the presence of common shock, a simple piece rate is suboptimal, but a tournament may dominate a piece-rate scheme which requires $w_{00}=w_c$. Because by filtering out the common shock, the tournament may expose agents to less risk than a piece-rate scheme.

Please note that the effectiveness of a tournament depends on three prerequisites: firstly, the agents are similar; Secondly, they are facing common risk shocks; Thirdly, there is no collusion. See Nie (2015) on *Journey to the West*.

3.1.4 Applications

When the old Bell System/AT&T selects its CEO of headquarter, it usually chooses the head of a regional company (*tiao*) rather than the head of functional HQ (*kuai*).

Why does a staff's salary increase rapidly after being promoted, while his ability is not substantially improved?

Is a tournament the competition among local governments in China?

References

Bo, Zhiyue, 1996, “Economic Performance and Political Mobility: Chinese Provincial Leaders,” *Journal of Contemporary China*, 5(12): 135-154.

Li, Hongbin and Li-An Zhou, 2005, “Political Turnover and Economic Performance: the Incentive Role of Personnel Control in China”, *Journal of Public Economics*, 89, 1743-1762.

陶然、苏福兵、陆曦、朱昱铭，2010，《经济增长能够带来晋升吗？——对晋升锦标赛理论的逻辑挑战与省级实证重估》，《管理世界》，第12期。

聂辉华，《跟<西游记>学创业——一本人人都要读的管理秘籍》，中国人民大学出版社，2015年。