3. Complete Contracts II: Static Multilateral Contracting

Huihua NIE
School of Economics
Renmin University of China
www.niehuihua.com
niehuihua(at)vip.163.com

3.2 Adverse Selection IV: Bilateral Trading

3.2.1 Coase Theorem and asymmetrical information

Coase Theorem (Coase, 1960) provides a fundamental framework to analyze contracts and organizations with symmetrical information. It is worth to point that Coase Theorem has some strict assumptions, e.g., income effect (Chipman, 1998). Students should understand "quasi-linear" utility function, say, $v^1(p,w^1,h)=\Phi^1(p,h)+w^1$, which is the key to model non-income effect and assure trade benefit. However, when one party of agents has private information, it raises the problem of economics of information, which considers all problems as how to allocate the power and information (rents).

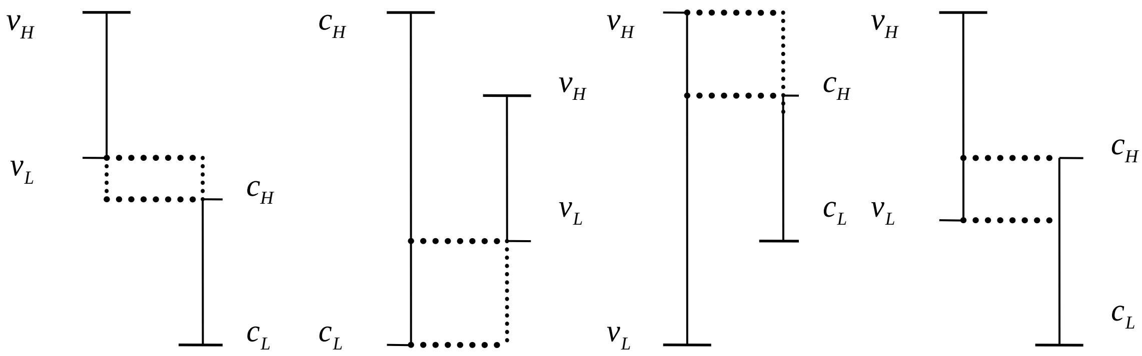
Under unilateral asymmetric information, say, a buyer has two possible valuations for a good: $v_H > v_L > 0$, and a seller has a product with cost of 0. Let $\beta = \Pr(v = v_H)$, then the (risk-neutral) seller's best offer to the buyer is $P = v_H$ whenever $\beta v_H \geq v_L$. [1] There is ex post inefficient trade with probability ($1-\beta$). Coase theorem doesn't hold in this case. This simple example highlights one obvious facet of the fundamental *trade-off between allocative efficiency and the distribution of information rent*. If instead the informed party (here, the buyer) has all bargaining power, then there is always ex post efficient trade ($P = 0$). However, if one party holds both power and private information, how to supervise he in reality? The separation of information and power is our fortunate in this time. As North (1981) said, dispersed resource leads to contractual state, but centralized resource leads to predatory state. In this sense, we can understand why we must give up central planned economy.

3.2.2 Bilateral trading: heuristic figures

But this solution is not available when there is bilateral asymmetric information. We show in this section that (*ex post*) efficient trade ($v \geq c$) can (almost) always be achieved if the parties' participation is obtained *ex ante*, before they learn their type, while it cannot be achieved if the parties' participation decision is made when they already know their type (*ex post*).

[1] If the quantity can be chosen, the pricing will be nonlinear in screen model.

Consider the situation where the seller has two possible costs $c_H > c_L \geq 0$, and the buyer's valuations are as follows. Both parties know their types.



(1) $v_H > v_L > c_H > c_L$

In the case efficient trade is guaranteed by fixing a price $P \in [c_H, v_L]$, whether binary or continuous.

(2) $c_H > v_H > v_L > c_L$

In the case efficient trade is guaranteed by setting a price $P \in [c_L, v_L]$ and letting the seller decide whether he wants to trade at that price. But **it is wrong** when value is continuous, and the real cost of the seller is above v_L and below the real valuation of the buyer, trade is efficient but can not be attained.

(3) $v_H > c_H > c_L > v_L$

In the case efficient trade is guaranteed by setting a price $P \in [c_H, v_H]$ and letting the buyer decide whether he wants to trade at that price. Also, efficient trade can't be attained when the value is continuous.

(4) $v_H > c_H > v_L > c_L$

In the case, efficient trade can not attain.

If the value is continuous, in the case (2), (3) and (4), efficient trade can attain only if *ex ante* individual-rationality constraints are relevant, not *interim* individual-rationality constraints are relevant. In sum, the efficient trade can not be assured if the costs and the values are crossed.

3.2.3 Efficient trade under ex ante individual-rationality constraints

Suppose that the principal (say, a social planner) offers the buyer and seller, *before* each one has learned his type, the following bilateral trading contract: $\mathbb{C} = \{P(v_i, c_j) \equiv P_{ij}; x(v_i, c_j) \equiv x_{ij}\}$, where we assume that $v_H > c_H > v_L > c_L$ and $x_{ij} = 1$ if $v_i \geq c_j$ (efficient condition), otherwise is 0. $\beta = \Pr(v = v_H)$, $\gamma = \Pr(c = c_L)$.

The seller's IC and ex ante IR constraints then take the form

(IC-S-L) $(1-\beta)(P_{LL} - c_L) + \beta(P_{HL} - c_L) \geq (1-\beta)P_{LH} + \beta(P_{HH} - c_L)$ (9-1)

(IC-S-H) $(1-\beta)P_{LH} + \beta(P_{HH} - c_H) \geq (1-\beta)(P_{LL} - c_H) + \beta(P_{HL} - c_H)$ (9-2)

(IR-S) $\gamma[(1-\beta)P_{LL} + \beta P_{HL} - c_L] + (1-\gamma)[(1-\beta)P_{LH} + \beta(P_{HH} - c_H)] \geq 0$ (9-3)

Notice that actually $x_{LH} = 0$, and there is no trade for P_{LH}.

Similarly, the buyer's IC and IR constraints are

(IC-B-L) $\gamma(v_L - P_{LL}) + (1-\gamma)P_{LH} \geq \gamma(v_L - P_{HL}) + (1-\gamma)(v_L - P_{HH})$ (9-4)

(IC-B-H) $\gamma(v_H - P_{HL}) + (1-\gamma)(v_H - P_{HH}) \geq \gamma(v_H - P_{LL}) + (1-\gamma)P_{LH}$ (9-5)

(IR-B) $\beta[v_H - \gamma P_{HL} - (1-\gamma)P_{HH}] + (1-\beta)[\gamma(v_L - P_{LL}) - (1-\gamma)P_{LH}] \geq 0$ (9-6)

In order to analyze the role of price in the trade, call $\bar{P}$ the expected payment the buyer will have to make to the seller:

$$\bar{P} = \gamma[(1-\beta)P_{LL} + \beta P_{HL}] + (1-\gamma)[(1-\beta)P_{LH} + \beta P_{HH}]$$

Then the two IR conditions (9-3), (9-6) can be rewritten as

$$\beta v_H + (1-\beta)\gamma v_L \geq \bar{P} \geq \gamma c_L + (1-\gamma)\beta c_H \quad (9\text{-}7)$$

This requirement implies a condition on the expected *level* of payments, which can be adjusted without any consequence on incentive constraints, which depend on the *differences* of payment across realization of costs and valuations. These incentive constraints can be redefined for the seller and buyer, respectively, as

$$(1-\beta)c_H \geq (1-\beta)(P_{LL} - P_{LH}) + \beta(P_{HL} - P_{HH}) \geq (1-\beta)c_L \quad (9\text{-}8)$$

and

$$(1-\gamma)v_H \geq \gamma(P_{HL} - P_{LL}) + (1-\gamma)(P_{HH} - P_{LH}) \geq (1-\gamma)v_L \quad (9\text{-}9)$$

If constraints (9-7)-(9-9) are satisfied, both parties are happy to participate and truth telling results, so that we have implemented the ex post efficient allocation. The fact that payments exist such that all constraints are satisfied is easy to see. Indeed, the constraints can be satisfied recursively (BD, p246-247). So, efficient trade can be achieved.

3.2.4 Inefficient trade under interim individual-rationality constraints

Now suppose that the buyer and seller know their type before signing the contract. Then the ex ante IR conditions must be replaced by the following conditions (like limited liability constraints):

For the seller:

(IR-S-L) $(1-\beta)P_{LL}+\beta P_{HL}-c_L\geq 0$ (9-10)

(IR-S-H) $(1-\beta)P_{LH}+\beta(P_{HH}-c_H)\geq 0$ (9-11)

For the buyer:

(IR-B-H) $v_H-\gamma P_{HL}-(1-\gamma)P_{HH}\geq 0$ (9-12)

(IR-B-L) $\gamma(v_L-P_{LL})-(1-\gamma)P_{LH}\geq 0$ (9-13)

We will prove that for some values of β and γ, these four conditions together with the IC conditions (9-1), (9-2), (9-4) and (9-5) cannot all simultaneously hold.

On the one side, when $\beta\to 1$, from (9-1) and (9-2), we have $P_{HH}\approx P_{HL}\equiv\hat{P}$. It means when the seller thinks the buyer is almost surely of type v_H, he will produce with probability close to 1 whatever his type and request a high price.

(9-10) and (9-11) can reduce to

$\hat{P}\geq c_L$ and $\hat{P}\geq c_H$

In words, $\hat{P}$ has to cover his cost, whether low or high.

On the other side, (9-5) can be rewrite as:

$v_H-\hat{P}\geq\gamma(v_H-P_{LL})-(1-\gamma)P_{LH}$ (9-14)

Applying $\beta\to 1$ and (9-14), (9-12) and (9-13) can be rewrite as:

$\hat{P}\leq v_H$ (9-15)

$\hat{P}\leq\gamma v_L+(1-\gamma)v_H=v_H-\gamma(v_H-v_L)$ (9-16)

where, $\gamma(v_H-v_L)$ is information rents of v_H buyer. We know (9-16) is more binding than (9-15). Hence, we can collect all interim IR conditions together, and have

$\gamma v_L+(1-\gamma)v_H\geq\hat{P}\geq c_H$ (9-17)

When $\gamma\to 1$, that is the seller has a low cost with probability close to 1, (9-17) does not hold. The reason behind it is because the buyer believes that he is almost certainly facing a low-cost seller, it becomes very attractive for him to pretend to have a low valuation, since in any case the probability of trade is almost 1. To prevent the buyer from pretending this, we need $\hat{P}\leq v_L$, but this is incompatible with $\hat{P}\geq c_H$. In other words, when all the parties have incentive to misreport their types, the information rents exceed the surplus from trade (BB constraint).

It is so-called **inefficiency theorem** (Myerson-Satterthwaite, 1983): Suppose that the seller's cost and the buyer's valuation have differentiable, strictly positive densities on [$\underline{c}$, $\bar{c}$] and [$\underline{v}$, $\bar{v}$], that there is a positive probability of gains from trade ($\underline{c} < \bar{v}$), and there is a positive probability of no gains from trade ($\bar{c} > \underline{v}$). Then there is no efficient trading outcome that satisfies IR, IC and BB constraints (Fudenberg-Tirole, 1991, p277).

It suggests that the Coase Theorem may break down in voluntary trading situations with multilateral asymmetric information. It is a version of Rawls' "the veil of ignorance" which we will discuss in incomplete contracts theory. An example of bonus points for sports and art majors during the college entrance examination.

It also points to the potential value of institutions with **coercive power** that can break interim participation constraints and secure participation at an ex ante stage. An example of China's New Rural Cooperative Medical Scheme.

3.2.5 Groves-Clark mechanism

In the lowest limit, we dislike coercive power by government. Fortunately, we have other way to attain efficient outcome. Sometimes if we can suppose that the agents' reservation utilities are arbitrarily low (which means we can omit participation constraints), efficient allocations can be implemented in *dominant strategies* by the Groves-Clark mechanism (Groves, 1973; Clarke, 1971).

The intuition behind the Groves-Clark mechanism is the same as following Vickrey auction. Suppose that the agents' preferences are $u_i = \theta_i x + t_i$, and then a typical public infrastructure problem is

$$x^*(\hat{\theta}) = 1 \text{ if } \sum_{j=1}^{I} \hat{\theta}_j \geq c \text{ or 0 otherwise}$$

$$t_i(\hat{\theta}) = \sum_{j \neq i} \hat{\theta}_j - c \text{ if } \sum_{j=1}^{I} \hat{\theta}_j \geq c \text{ or 0 otherwise}$$

For instance, there is a bridge to build, whose costs are 100, and 5 persons in the crop. We have at least two mechanisms:

$M_1 : (20, \sum_{i=1} m_i \geq 100)$, which will be inefficient because of inefficiency theorem.

$M_2 : (100 - \sum_{j \neq i} m_j , \sum_{i=1} m_i \geq 100)$, in which truth-telling is a weakly dominating strategy for agent i . His payoff is $\pi_i = \theta_i - 100 + \sum_{j \neq i} m_j$. But, when $\sum_{j \neq i} m_j \geq 100$, agent i will get positive subside from the government, which can break the budget balance.

Let's show that by another example by Prof. WANG Hao. A college dorm has 8 students with the even distribution on [200, 400] on buying a television of ￥2000. What's the transfer of a student? 0.5*(200+400)*7-2000=100, so the college must subsidize ￥100 for every student!

The Groves-Clark mechanism is extended to AGV mechanism by d'Aspremont and Gerard-Vare (1979), which is satisfied by BB constraint and is implementable in Bayesian equilibrium but not in dominant strategies equilibrium. Cramton-Gibbons-Klemperer (1987) show that if the initial shares are fairly evenly distributed, there exists efficient mechanisms that satisfy IC, IR and BB.