2. Complete Contracts I: Static Bilateral Contracting

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2.4 Adverse Selection II: Signaling

2.4.1 Spence model: education market

Nowadays, "involution"（内卷）[1] has become the most popular workplace phenomenon. Why and how it happens? We follow the simplest version of Spence (1973, 1974). There are a worker and a firm. The worker's productivity can be either r_H or r_L, with $r_H > r_L > 0$. Let β_i be the firm's prior belief that $r = r_i$. A worker of type $i = L, H$ can get e years of education at cost $c(e) = \theta_i e$ before entering the labor market. The key assumption is that $\theta_H < \theta_L$, and education has nothing to do with productivity. The worker's utility function is $u_i = r_i - \theta_i e$. Obviously, the setting satisfies S-M condition ($\frac{\partial}{\partial \theta}(\frac{dr}{de}) = \frac{\partial}{\partial \theta}[-\frac{\partial u / e}{\partial u / r}] = \frac{\partial}{\partial \theta}(\theta_i) = 1 \neq 0$). Suppose that the worker (agent[2]) has all the bargaining power over the firm (principal). Recall section 2.3.5. The timing is as figure 5-1.

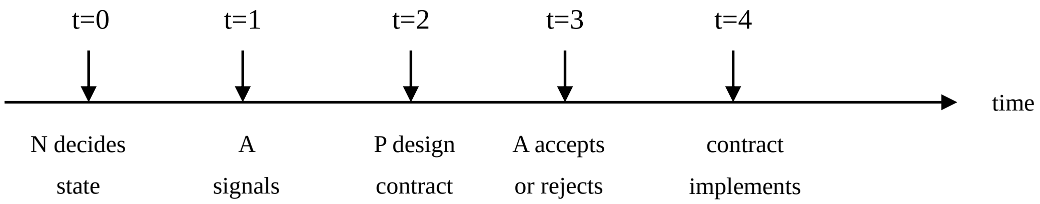

Fig. 5-1

It is a dynamic game with incomplete information, and its solution is perfect Bayesian equilibrium (PBE). PBE＞SE＞trembling-hand equilibrium.

Under symmetric information, the first-best solution is $e_H = e_L = 0$ and $w_i = r_i$. Under asymmetric information for productivity θ_i, let $p_i(e)$ denote the probability that the worker of

[1] The term "involution" was originated from the research of cultural anthropologist Clifford Geertz on Indonesian agriculture and was introduced to China by Professor Philip C. C. Huang.
[2] The worker is a principal in BD.

type i chooses education level e, and $\beta(\theta_i \mid e)$ for the firm's revised beliefs about productivity upon observing e.

We solve a typical dynamic game with incomplete information by three steps: first, we guess conditional beliefs $\beta(\theta_i \mid e)$ for principal; second, we conclude agent's best response $p_i(e)$; third, we check whether the beliefs $\beta(\theta_i \mid e)$ are consistent with Bayesian rule, which is the key.

In this case, PBE satisfies four conditions:

(1) Worker's IC condition: $e^* \in \arg\max_e [\beta(\theta_H \mid e) r_H + \beta(\theta_L \mid e) r_L - \theta_i e]$.

(2) Firms pay workers their expected productivity: $w(e) = \beta(\theta_H \mid e) r_H + \beta(\theta_L \mid e) r_L$.

(3) Firm's posterior beliefs (Bayes' rule): $\beta(\theta_i \mid e) = \dfrac{p(\theta_i) p(e \mid \theta_i)}{p(e)} = \dfrac{p(\theta_i) p(e \mid \theta_i)}{\sum_{j=1}^{2} p(\theta_j) p(e \mid \theta_j)}$.

An example of hot pepper for Sichuan Chinese: if you see someone eating hot pepper, what is the probability of Sichuan Chinese for this guy?

$$\beta(\text{四川人} \mid \text{吃辣椒}) = \frac{p(\text{四川人}) p(\text{吃辣椒} \mid \text{四川人})}{p(\text{四川人}) p(\text{吃辣椒} \mid \text{四川人}) + p(\text{其他人}) p(\text{吃辣椒} \mid \text{其他人})}$$

$$= \frac{0.1 \times 0.9}{0.1 \times 0.9 + 0.9 \times 0.4} = 20\%$$

(4)Posterior beliefs are otherwise not restricted: if $p_i(e) = 0$ for $i = L, H$ (such that $\sum_{i=1}^{2} \beta_i p_i(e) = 0$, and Bayes' rule gives no prediction for posterior beliefs), then $\beta_i(\theta_i \mid e)$ can take any value in [0,1]. We must give exogenous belief on off-equilibrium path.

In signaling games the difficulty is usually not to find a PBE. Rather, the problem is that there exist too many PBEs.

(1) **Separating PBE**s, where different types choose different signal in equilibrium: $e_H \neq e_L$ such that $\beta(\theta_H \mid e_H) = \dfrac{\beta_H p_H(e_H)}{\beta_H p_H(e_H) + \beta_L p_L(e_H)} = \dfrac{\beta_H \times 1}{\beta_H \times 1 + \beta_L \times 0} = 1$, $\beta(\theta_L \mid e_L) = 1$, $\beta(\theta_H \mid e_o) = 0$ if $e_O \neq e_H$ (off-equilibrium path belief), and $e_L = 0$, $w_i = r_i$.

Proof.

(ICH) $r_H - \theta_H e_H \geq r_L \Rightarrow e_H \leq \dfrac{r_H - r_L}{\theta_H}$

(ICL) $r_L \geq r_H - \theta_L e_H \Rightarrow e_H \geq \dfrac{r_H - r_L}{\theta_L}$

$$\therefore S_S = \{(e_H, e_L) \mid e_L = 0 \text{ and } e_H \in [\frac{r_H - r_L}{\theta_L}, \frac{r_H - r_L}{\theta_H}]\}.$$

(Here we omit IR condition, which is $u_i = r_i - \theta_i e_i \geq 0 \Rightarrow e_i \leq \frac{r_i}{\theta_i}$. Surely it can be satisfied for H and L, because $e_H \leq \frac{r_H - r_L}{\theta_H} < \frac{r_H}{\theta_H}$ based on ICH, and $e_L = 0 < \frac{r_H}{\theta_H}$.)

(2) **Pooling PBE**s, where each type of agent chooses the same signal in equilibrium: $e_H = e_L = e_p$ so that the belief on out-of-equilibrium path is $\beta(\theta_H \mid e) = \beta_H$ and $\beta(\theta_L \mid e) = \beta_L$ whenever $e \geq e_p$, and $\beta(\theta_H \mid e) = 0$ otherwise, where $w(e_p) = \beta_H r_H + \beta_L r_L \equiv \bar{r}$.

Proof.

(ICL) $\bar{r} - \theta_L e_p \geq r_L \Rightarrow e_p \leq \frac{\beta_L r_L + \beta_H r_H - r_L}{\theta_L}$

$$\therefore S_P = \{(e_H, e_L) \mid e_L = e_H = e_p \text{ and } e_p \in [0, \frac{\beta_L r_L + \beta_H r_H - r_L}{\theta_L}]\}.$$

(Here we omit ICH condition, i.e. $\bar{r} - \theta_H e_p \geq r_L - \theta_H e_o$, where $e_o \neq e_p$.)

(3) Semiseparating (hybrid) PBEs, where at least one type of agent is mixing between two signals, one of which is also chosen with positive probability by the other type of agent. It implies that

$$r_H - \theta_H (\frac{r_H - r_L}{\theta_L}, \frac{r_H - r_L}{\theta_H}) = \bar{r} - \theta_H \frac{\beta_L r_L + \beta_H r_H - r_L}{\theta_L}$$

or $r_L = \bar{r} - \theta_L \frac{\beta_L r_L + \beta_H r_H - r_L}{\theta_L} \Rightarrow (1 - \beta_L) r_L = \beta_H r_H$.

Example

Bachelor degree is separating equilibrium, Master degree is hybrid equilibrium, and Ph.D. degree is pooling equilibrium.

2.4.2 Refinements

Spence: social custom or conventions; Aumann: correlated equilibrium; Schelling: focal points.

Fudenberg-Tirole (1991): adventure in this topic by introducing restrictions on the set of allowable off equilibrium path

Cho-Kreps intuitive criterion

The most popular refinement is Cho-Kreps (1987).

DEFINITION Cho-Kreps intuitive criterion: Let $u_i^* = w_i^*(e_i) - \theta_i e_i$ denote the equilibrium payoff of type i. Then, $\beta(\theta_j \mid e) = 0$ for $e \neq (e_i, e_j)$, whenever $r_H - \theta_j e < u_j^*$ and $r_H - \theta_i e \geq u_i^*$ ($i = L, H$; $i \neq j$).

The intuition behind the definition is that when a deviation is dominated for one type of player but not the other one, this deviation should not be attributed to the player. Here, a deviation is dominated for type θ_j but not θ_i, so $\beta(\theta_j \mid e) = 0$. When $e \geq e_d$, we have $w_H = r_H$, $\beta(\theta_H \mid e) = 1$ and $\beta(\theta_L \mid e) = 0$.

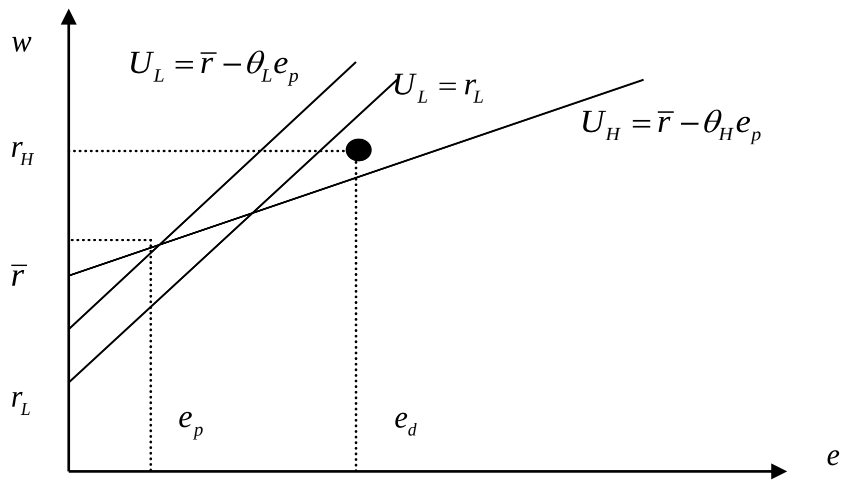

Fig. 5-2

Applying this test to pooling equilibria. Type θ_H may find a profitable deviation, by, say, increasing his education level to e_d, which is dominated for type θ_L. In this case, we can discard all pooling equilibria, as figure 5-2; likely, we can discard all semiseparating equilibria.

Furthermore, applying this test to separating equilibria, one eliminates all but one equilibrium, the so-called "least-cost" separating equilibrium in which $e_L = 0$ and $e_H = \frac{r_H - r_L}{\theta_L}$, that is a pure-strategy equilibrium.

As plausible as the Cho-Kreps intuitive criterion may be, it does seem to predict implausible equilibrium outcomes in some situations. For example, in separating equilibria, suppose now β_L is arbitrarily small ($\beta_L = \delta \to 0$). The cost of separating is $c(e_H; \theta_H) = \theta_H \frac{r_H - r_L}{\theta_L}$, but the benefit of separating is $\Delta w = r_H - [(1-\delta) r_H + \delta r_L] = \delta (r_H - r_L) \to 0$. It is too expensive for society!

Maskin-Tirole's approach

If workers offer the firm a contingent contract before the choice of the signal, the problem of inefficient signal can be eliminated. The timing now is as figure 5-3.

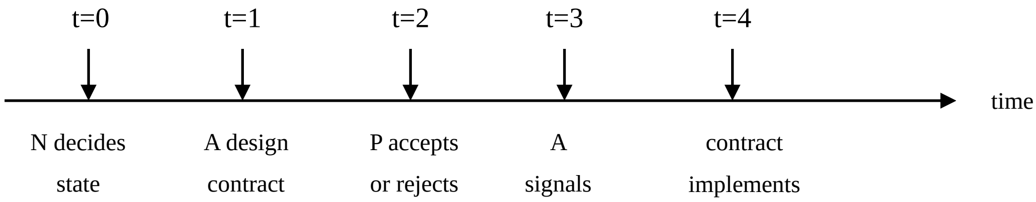

Fig. 5-3

(1) Suppose $\bar{r} = r_L \beta_L + r_H \beta_H$, which under there is nobody to be educated and the society saves signaling costs.

(2) If $\bar{r} \le r_H - \theta_H \frac{r_H - r_L}{\theta_L}$, $w(e) = \begin{pmatrix} r_H, e \ge \frac{r_H - r_L}{\theta_L} \\ r_L, otherwise \end{pmatrix}$. It is ex post efficient for θ_H and P in separating equilibrium. At this time deviation is profitable.

(3) If $\bar{r} > r_H - \theta_H \frac{r_H - r_L}{\theta_L}$, $w(e) = \bar{r}$, and it is ex post efficient for both θ_H and θ_L in pooling equilibrium.

2.4.3 Applications

Monopoly pricing: Milgrom-Roberts (1986); Pecking-order theory of corporate finance: Myers-Majluf (1984); A spell of unemployment in labor market: Ma-Weiss (1993).

2.4.4 Cheap talk

Under some circumstances agent can talk to principal without any cost, and private-information problem will be solved by cheap talk. There is an example by Farrel-Rabin (1996, JEP).

	Rayco company's job		
Sally's ability		Demanding	Undemanding
	High	2,1	0,0
	Low	0,0	3,1