2. Complete Contracts I: Static Bilateral Contracting

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2.3 Adverse Selection I: Screening

2.3.1 Typical case: lemon market

Chinese idiom: 滥竽充数; Akerlof (1970): market for used cars. And also Mirrlees (1971) formally analyzed the problem of hidden information.

Timing: *Ex ante* adverse selection

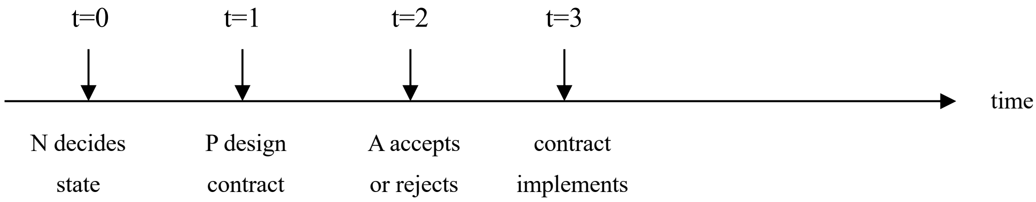

Fig. 1-3

☛ What's the type of this game theory, **static** game with imperfect information or dynamic game with imperfect information? Beyesian equilibrium: if $\sum_i p_i(a^*,\theta_i,\theta_{-i}) \succeq_i \sum_i p_i(a',\theta_i,\theta_{-i})$, agents will choose a^*.

2.3.2 Basic model: nonlinear pricing

Consider a transaction between a buyer (customer) and a seller (Telcom), where the seller doesn't know perfectly how much the buyer is willing to pay for his good. Actually it is a problem of "second-degree price discrimination". And "all competition is discriminatory" (Alchian-Allen, 1972).

The buyer's preferences is $u(q,T,\theta)=\theta u(q)-T$, where $u(0)=0$, $u'(q)>0$, and $u''(q)<0$. $\theta\in\{\theta_L,\theta_H\}$ is private information for buyer, and seller knows only the distribution of θ, $f(\theta)$. The buyer is of type θ_L with probability $\beta\in[0,1]$ and of type θ_H with probability $1-\beta$. The seller's preference is $\pi=T-cq$. So, how does the seller design the terms of the contract?

(1)Benchmark: Perfect price discrimination

To begin with, suppose that the seller if perfectly informed about the buyer's characteristics, he will offer a type-specific contract (T_i, q_i) for type θ_i ($i = L, H$). Formally, the seller will solve

$$\underset{T_i, q_i}{Max}\, T_i - cq_i$$

s.t. (IR) $\theta_i u(q_i) - T_i \geq \underline{U}$

Solve two equations, we get $\theta_i u'(q_i) = c$ and $\theta_i u(q_i^{FB}) = T_i + \underline{U} \Leftrightarrow T_i = -\underline{U} + \theta_i u(q_i^{FB})$.

The total profit of the seller is $\beta(T_L - cq_L) + (1-\beta)(T_H - cq_H)$.

[Note 1] By having the buyer select a quantity such that marginal utility equals marginal cost (*first best condition*), meanwhile by binding IR constraint via T the seller appropriates the full surplus and leaves no rent to the buyer.

[Note 2] The best contract is implemented by type-specific two-part tariffs.

(2) Second best: optimal nonlinear pricing

With asymmetric information, type θ_H will pretend to be θ_L and get information rent (see below), so the seller will fail to perfectly price discriminate. An illustration is $\theta_h u(q_l) - T_l \geq \underline{U}$ and $\theta_l u(q_l) - T_l \geq \underline{U}$ can not hold simultaneously. What can he do? Without loss of generality, the optimal price can be described as a schedule $(q_i, T(q_i))$. The problem is

$$\underset{T(q)}{Max}\, \beta[T(q_L) - cq_L] + (1-\beta)[T(q_H) - cq_H]$$

s.t. (IR) $\theta_i u(q_i) - T(q_i) \geq \underline{U}$

(IC) $q_i \in \arg\underset{q}{max}\, \theta_i u(q_i) - T(q_i)$ for $i = L, H$

Also, we can treat $\underline{U}$ as exogenous and normalize it to be 0. Notice that the problem involves optimization over a schedule $T(q_i)$ under IC condition that involves optimization over q. We can solve the problem step-by-step as ***5*** steps:

Step 1: Apply the revelation principle

Background. Social choice $\rightarrow$ mechanism design $\rightarrow$ revelation principle $\rightarrow$ direct revelation mechanism/truth-telling implementation.

With revelation principle, we can restrict each schedule $T(q_i)$ to the pair of optimal choices made by the two types of buyers $\{[q_L, T(q_L)]$ and $[q_H, T(q_H)]\}$. We define $T(q_i) \equiv T_i$, then the problem can be rewritten as

$$\underset{T_i, q_i}{Max}\, \beta(T_L - cq_L) + (1-\beta)(T_H - cq_H)$$

s.t.

(IRL) $\theta_L u(q_L) - T_L \geq \underline{U}$ ✓

(IRH) $\theta_H u(q_H) - T_H \geq \underline{U}$

(ICL) $\theta_L u(q_L) - T_L \geq \theta_L u(q_H) - T_H$

(ICH) $\theta_H u(q_H) - T_H \geq \theta_H u(q_L) - T_L$ ✓

☛ Why we need not revelation principle in moral hazard problem?

Step 2: Eliminate the "high" type's participation constraint.

$\because \theta_H > \theta_L$ and ICH, $\therefore \theta_H u(q_H) - T_H \geq \theta_H u(q_L) - T_L > \theta_L u(q_L) - T_L \geq \underline{U}$.

Step 3: Check Spence-Mirrlees single-crossing condition, and eliminate the "low" type's incentive compatibilities constraint.

The S-M condition requires that the marginal utility of consumption vary with type, such that a separating equilibrium exits. The S-M condition is

$$\frac{\partial}{\partial\theta}\left[\frac{dT}{dq}\right] = \frac{\partial}{\partial\theta}\left[-\frac{\partial u/\partial q}{\partial u/\partial T}\right] = \frac{\partial}{\partial\theta}\left(\frac{-\theta u'(q)}{-1}\right) = u'(q) > 0$$

[Note] We need S-M condition $\neq 0$, not necessarily >0 (for example θ stands for costs).

S-M condition implies the monotonicity condition. Adding ICH to ICL, we can get $(\theta_H - \theta_L)[u(q_H) - u(q_L)] \geq 0$, which means that $q_H \geq q_L$.($\because \theta_H > \theta_L$). That is monotonicity and implementation condition. By S-M condition we guess that the low type has no incentive to imitate high type, i.e., ICL is not binding. Specifically, because IRL is binding (and IRH is not), $T_L = -\underline{U} + \theta_L u(q_L)$, $-\underline{U} + \theta_L u(q_H) > -\underline{U} + \theta_L u(q_L)$. We will check it latter.

Step 4: Check that the two remaining constraints of the relaxed problem will bind at the optimum.

$$\underset{T_i, q_i}{Max}\, \beta(T_L - cq_L) + (1-\beta)(T_H - cq_H)$$

s.t.

(IRL) $\theta_L u(q_L) - T_L \geq \underline{U}$

(ICH) $\theta_H u(q_H) - T_H \geq \theta_H u(q_L) - T_L$

If ICH is not binding, the seller will raise T_H until it does bind and improve his welfare. So does IRL. Also, we can check the λ and μ by Kuhn-Tucker theorem.

Step 5: Insert IRL and ICH constraints into seller's program, and perform the unconstrained optimization problem.

$$\underset{q_L, q_H}{Max}\, \beta[\theta_L u(q_L) - cq_L - \underline{U}] + (1-\beta)[\theta_H u(q_H) - cq_H - \underline{U} - (\theta_H - \theta_L)u(q_L)]$$

FOC:

$$\theta_H u'(q_H^*) = c\,,\ \ \theta_L u'(q_L^*) = \frac{c}{1-(\frac{1-\beta}{\beta}\frac{\theta_H - \theta_L}{\theta_L})} \ge c\text{[1]};$$

$$T_L = \theta_L u(q_L) - \underline{U}\,,\ \ T_H = \theta_H u(q_H) - \underline{U} - (\theta_H - \theta_L)u(q_L)\,,\ \ u^H = (\theta_H - \theta_L)u(q_L)\,.$$

It implies that $u'(q_H^*) < u'(q_L^*) \Rightarrow q_H^* > q_L^*$.

$\because$ ICH binds, we have $\theta_H u(q_H^*) - T_H^* = \theta_H u(q_L^*) - T_L^*$ or $\theta_H [u(q_H^*) - u(q_L^*)] = T_H^* - T_L^*$.

Because $q_H^* > q_L^*$ and $u(q_H^*) > u(q_L^*)$, $T_H^* > T_L^*$. So with $\theta_L < \theta_H$, we naturally have $\theta_L [u(q_H^*) - u(q_L^*)] < T_H^* - T_L^*$, which is $\theta_L u(q_H^*) - T_H^* < \theta_L u(q_L^*) - T_L^*$. That is ICL that is omitted constraint.

[Note 1] The low type has no consumer surplus.

[Note 2] The last part of the second brackets high type's *information rent*, which come from the fact that he can "mimic" the behavior of the other of type and increases with q_L.

Conclusions:

- IR condition binds only for low type (bad type), so low type gets zero rent, but high type (good type) get positive information rent which depends on q_L^*.
- IC condition binds only for high type not for low type, which means that high type's second-best consumption is the same as the first-best consumption ($q_H^* = q_H^{FB}$), that is "*no distortion at the top*". Low type's second-best consumption is lower than the first-best consumption ($q_L^* \le q_L^{FB}$), which is distorted. To do so, high type has no incentive to pretend to be low type.
- The point is that the principal should tradeoff between incentives and rent extraction.
- $\beta \downarrow$, $q_L^* \downarrow$ (if $q_L^* > 0$); $\beta \uparrow$, $q_L^* \uparrow$; $\beta = 0$, $q_L^* \ll q_L^{FB}$; $\beta = 1$, $q_L^* = q_L^{FB}$.

[1] If this expression is not positive, then the optimal solution involves $q_L^* = 0$.

☛ What happen if different types have different perverse utility? See LM (2000, ch3.4).

2.3.4 Applications

Second-degree price discrimination; Regulation and menu of contracts (LT, 1993), credit rationing (Stiglitz-Weiss, 1981), Optimal taxation (Mirrlees, 1971)

2.3.5 When principles compete for agents *

In the previous subsection, we assume that P has all the bargaining power, it is not necessarily true. Sometime when principles compete for agents, all the results may be inversed. Examples: Enterprises bosses competing for workers after Chinese Spring Festival since 2010; Chine Mobile and Unicom compete for iPhone of Apple.

Suppose that there are two types of agents, G and B. When agents work hard for a project (there is no room for moral hazard problem), G succeeds with probability p^G, and p^B for B, and $p^G > p^B$. If the project succeed, outcome performance is x_S, and agents get wage w_S; if the project fail, they are x_F and w_F, respectively.

There some risk-neutral principals compete for agents, and principals' expected profit is

$$EV = px_S + (1-p)x_F - pw_S - (1-p)w_F$$

Agents are risk averse, and their expected utilities are

$$EU^G = p^G u(w_S) + (1-p^G)u(w_F) \text{ and } EU^B = p^B u(w_S) + (1-p^B)u(w_F).$$

Notice that there is no effort cost, neither is moral hazard problem. We don't care what the optimal contract a P offers to a A, but what a series of optimal *equilibrium* contracts many Ps offer to different As.

(1) Benchmark: symmetric information

Let the typical type is T ($T = G, B$). Equilibrium contracts $C^T \equiv (w_S^T, w_F^T)$ must satisfy two conditions: P's expected profit is zero, that is

$$w^T = p^T x_S + (1-p^T)x_F \quad (4\text{-}1)$$

And contracts C^T is efficient, that is

$$\max_{w_S, w_F} p^T x_S + (1-p^T)x_F - p^T w_S - (1-p^T)w_F \quad (4\text{-}2)$$

s.t. $p^T u(w_S) + (1-p^T)u(w_F) \geq EU^T$

Solve expression (4-2) with Kuhn-Tucker theorem, we get

$$\frac{1}{u'(w_S)} = \frac{1}{u'(w_F)} = \lambda \Rightarrow w_S^T = w_F^T \tag{4-3}$$

Liking previous analysis, the first-best contract with symmetric information should provide full assurance for risk-averse agents. Together with expression (4-1), we have

$$w_S^T = w_F^T = p^T x_S + (1-p^T)x_F \tag{4-4}$$

Specifically, optimal equilibrium contracts are

$$w_S^G = p^G x_S + (1-p^G)x_F\,,\ \ w_S^B = p^B x_S + (1-p^B)x_F$$

We can show C^T as figure 4-1:

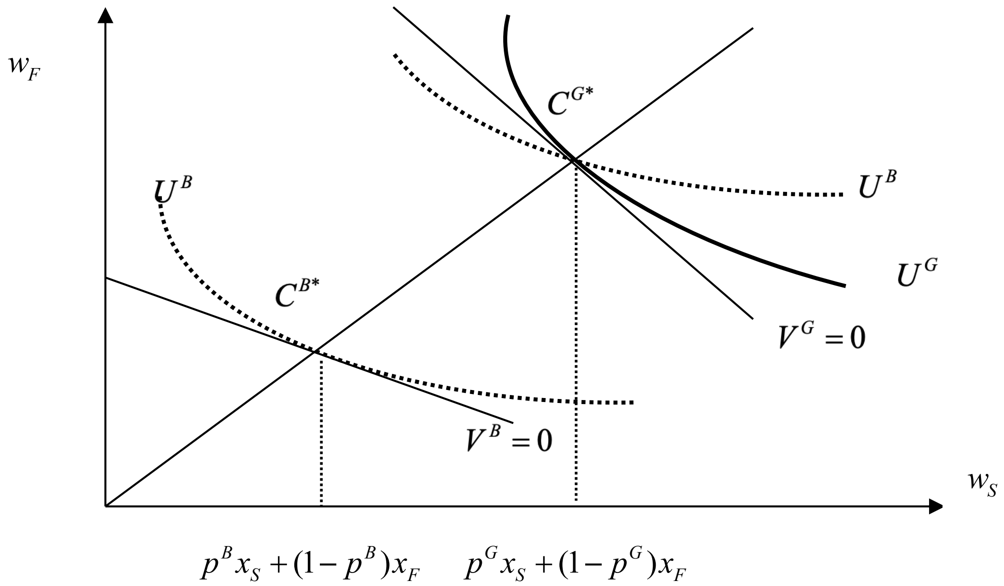

Fig. 4-1

[Note 1] G's indifference curve is more sloping than that of B, which implies that G will succeed with larger probability. Because

$$MRS = -\frac{dw_F}{dw_S}\,|\,EU^G = \frac{\partial U^G/\partial w_S}{\partial U^G/\partial w_F} = \frac{p^G u'(w_S)}{(1-p^G)u'(w_F)} > \frac{p^B u'(w_S)}{(1-p^B)u'(w_F)} = -\frac{dw_F}{dw_S}\,|\,EU^B$$

[Note 2] The optimal equilibrium contracts are on the 45° line.

[Note 3] With asymmetric information, all the B will have incentive to pretend to be G (as Fig. 4-1), and get high wage package. Then P will go bankrupt.

(2) Pooling equilibrium

If a pooling equilibrium exist, it must satisfy expression (4-1), and on the line

$$V^T = p^T x_S + (1-p^T)x_F - p^T w_S - (1-p^T)w_F = 0$$

Where $p^T = qp^G + (1-q)p^B$, and q is the probability of type G.

If the pooling equilibrium C^T exists in which both G and B get the same payment, it must on the cross of indifference curves of G and B, and principals' zero profit budget line, as figure 4-2.

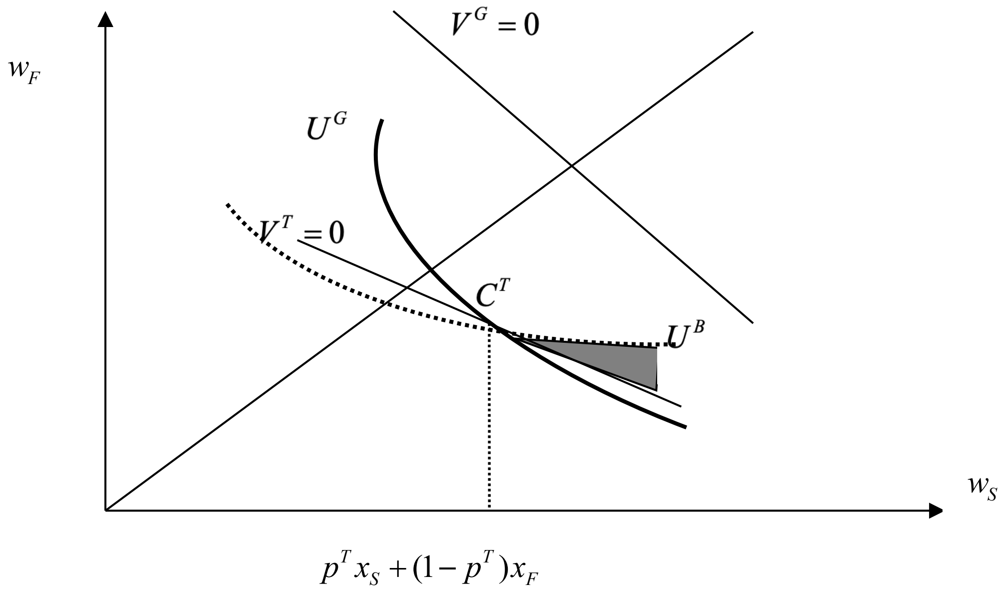

Fig. 4-2

Notice that if P offer a new contract $C^{T'}$ in the shade area of Fig. 4-2, G will accept for his utilities rise, and P will get positive profit ($V^T < V^G$), but B will get less benefit. By doing so, $C^{T'}$ is Pareto efficient for G and P, which leads to B will be screened. Hence, C^T is impossibly pooling equilibrium. In a word, pooling equilibrium doesn't exist. Why? Because S-M condition!

(3) Separating equilibrium

Remember that the problem is that B want to pretend to be G, which is contrary to previous subsection. At first, we can find out a contract C^{B*} which is on the line $V^B = 0$ and is Pareto efficient. Secondly, we find out a contract C^G which is on the line $V^G = 0$ and on B's indifference curve. Equilibrium contracts pair (C^G, C^{B*}) is shown as figure 4-3.

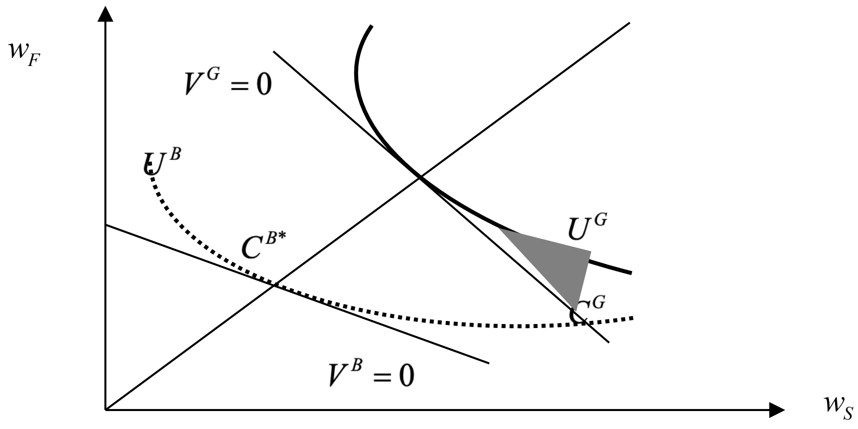

Fig. 4-3

Technically, C^G is defined by following expressions:

$$u(w^{B*}) = p^B u(w_S^G) + (1-p^B)u(w_F^G) \tag{4-5}$$

$$p^G x_S + (1-p^G)x_F - p^G w_S + (1-p^G)w_F = 0 \tag{4-6}$$

Expression (4-5) indicates that there is no arbitrage advantage for B type (IC condition), and (4-6) means that C^G is also on zero profit line. Actually, it must be above the line $V^T = 0$, otherwise principals will get positive profit, and G type will improve, such that (C^G, C^{B*}) is not an equilibrium, as the shade area of figure 4-4.

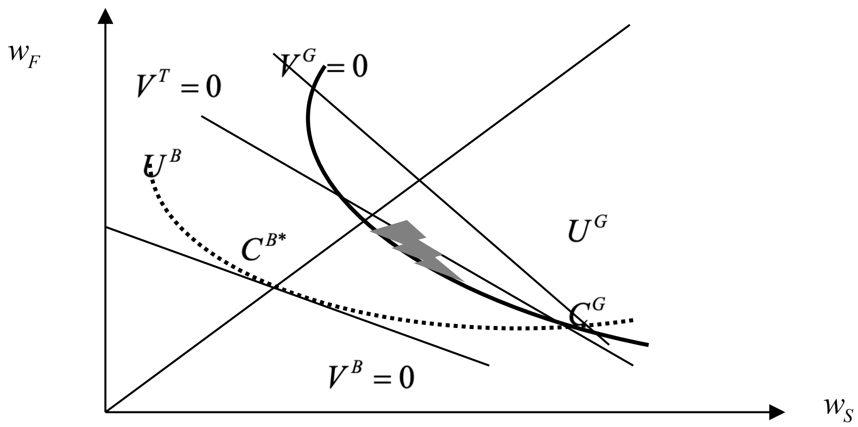

Fig. 4-4

In fact, the less q, the more down line $V^T = 0$. So, in order to assure that separating equilibrium exists, we hope that q is sufficiently small. To make the optimal wage for G type is less attracting for B type, P must decrease the wage for G type. When G type is sufficiently enough, P will get profitable.

Applications: the salary for government official. How about university faculty?

(4) Conclusions

- If $q \le \overline{q}$, (C^G, C^{B*}) is a separating equilibrium;
- If $q > \overline{q}$, equilibrium does not exist;
- In equilibrium state, the lowest efficient types get his utility as that under symmetric information, but the highest efficient types get less utility than that under symmetric information $(C^G < C^{G*})$. The loss of G type is the shade area of the Fig. 4-3. In this sense, it is contrary to simple adverse selection problem. However, the property that "top component is not distorted" also holds. Differently, in this subsection the top component is B type that has incentive to pretend to be other types as the highest efficient type in simple adverse selection problem.
- Notice that sometimes θ stands for contrary meaning, so we have contrary monotonicity, MLRP and S-M conditions.

2.3.6 Finite number of types

Different from previous subsections, now we suppose that there are at least three different preference types: $\theta_n > \theta_{n-1} > ... > \theta_1$ with $n \ge 3$. Also, the buyer has a utility function $u(q,T,\theta_i) = \theta_i u(q) - T$, and β_i is the proportion of buyers of type θ_i in the population. We can solve the problem step-by-step as ***6*** steps:

Step 1: Apply the revelation principle

$$\underset{T_i, q_i}{Max} \sum_{i=1}^{n} \beta_i (T_i - cq_i)$$

s.t.

(IR) $\theta_i u(q_i) - T_i \ge \underline{U}$ for all i

(IC) $\theta_i u(q_i) - T_i \ge \theta_i u(q_j) - T_j$ for all i, j

Step 2: Eliminate the "higher" types' participation constraints.

$\because \theta_i > \theta_1$, $\therefore \theta_i u(q_i) - T_i \ge \theta_i u(q_1) - T_1 > \theta_1 u(q_1) - T_1 \ge \underline{U}$.

Step 3: Check Spence-Mirrlees single-crossing condition and monotonicity.

The S-M condition is

$$\frac{\partial}{\partial\theta}[\frac{dT}{dq}] = \frac{\partial}{\partial\theta}[-\frac{\partial u/\partial q}{\partial u/\partial T}] = \frac{\partial}{\partial\theta}(\frac{-\theta_i u'(q_i)}{-1}) = u'(q_i) > 0$$

Summing the incentive constraints for types $\theta_i \ne \theta_j$, that is

$\theta_i u(q_i) - T_i \geq \theta_i u(q_j) - T_j$ and $\theta_j u(q_j) - T_j \geq \theta_j u(q_i) - T_i$

We have $(\theta_i - \theta_j)[u(q_i) - u(q_j)] \geq 0$. $\because \theta_i > \theta_j$, $u'(q) \geq 0$, $\therefore q_i \geq q_j$. That is, consumption must be monotonically increasing in θ when SM condition holds.

Step 4: Check local downward incentive constraints (LDICs) hold.

Consider the three types $\theta_{i+1} > \theta_i > \theta_{i-1}$. We have

(IC1) $\theta_{i+1} u(q_{i+1}) - T_{i+1} \geq \theta_{i+1} u(q_i) - T_i$ and (IC2) $\theta_i u(q_i) - T_i \geq \theta_i u(q_{i-1}) - T_{i-1}$.

$\because q_i \geq q_{i-1}$, $\therefore$ (IC2) implies that $\theta_{i+1} u(q_i) - T_i \geq \theta_{i+1} u(q_{i-1}) - T_{i-1}$. Together with (IC1), we get $\theta_{i+1} u(q_{i+1}) - T_{i+1} \geq \theta_{i+1} u(q_{i-1}) - T_{i-1}$ (IC3). Therefore, if for each type θ_i, the incentive constraint with respect to type θ_{i-1} --in other words, LDIC is satisfied—then all other downward incentive constraints are also satisfied if the monotonicity condition $q_i \geq q_{i-1}$ holds.

Step 5: Check the relaxed constraints bind at the optimum.

Use contradiction to prove. So the seller's problem reduces to

$$\underset{T_i, q_i}{Max} \sum_{i=1}^{n} \beta_i (T_i - cq_i)$$

s.t.

(IR) $\theta_1 u(q_1) - T_1 = \underline{U}$

(IC) $\theta_i u(q_i) - T_i = \theta_i u(q_{i-1}) - T_{i-1}$ for all i

(Monotonicity) $q_i \geq q_j$ where $\theta_i > \theta_j$

Step 6: Solving the reduced program.

The standard procedure for solving this program is first to solve the relaxed problem without the monotonicity condition and then to check whether the solution to this relaxed problem satisfies the monotonicity condition.

Lagrangian equation is

$$L = \sum_{i=1}^{n} \{\beta_i (T_i - cq_i) + \lambda_i [\theta_i u(q_i) - T_i - \theta_i u(q_{i-1}) + T_{i-1}]\} + \mu[\theta_1 u(q_1) - T_1 - \underline{U}]$$

FOC (for $1 < i < n$):

$$\frac{\partial L}{\partial q_i} = \lambda_i \theta_i u'(q_i) - \lambda_{i+1} \theta_{i+1} u'(q_i) = c\beta_i \Rightarrow u'(q_i) = \frac{c\beta_i}{\lambda_i \theta_i - \lambda_{i+1} \theta_{i+1}} > \frac{c\beta_i}{\lambda_i \theta_i}$$

$$\frac{\partial L}{\partial T_i} = \beta_i - \lambda_i + \lambda_{i+1} = 0$$

FOC (for $i = n$):

$$\frac{\partial L}{\partial q_n} = \lambda_n \theta_n u'(q_n) = c\beta_n$$

$$\frac{\partial L}{\partial T_n} = 0 \Leftrightarrow \beta_n = \lambda_n$$

Thus, we have $\theta_n u'(q_n) = c$, and $q_i < q_n$ for all the $i < n$. It satisfies the monoticity condition. In other words, consumption is efficient for $i = n$, and all types other than $i = n$ under-consume in equilibrium.

2.3.7 Applications

Job market (Spence, 1973, QJE)---next section

Career concern (Holmstrom, 1999, RES). A risk-neutral manager operating in a competitive labour market and his wage equals output. No contingent contracts can be made, so the manager is paid for his services in advance. Production function: $y_t = \eta + a_t + \varepsilon_t$; utility function: $U(w,a) = \sum_{t=1}^{\infty}[w_t - c(a_t)]$. η is manager's talent that is fixed and incompletely known to the manager and the market. η and ε are normally distributed with mean 0 and σ_η^2, σ_ε^2. For two periods, under rational expectation with $\tau = \dfrac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\varepsilon^2}$ (uncertainty), we have

$w_1 = E(y_1) = E(a_1) = \bar{a}_1$ (equilibrium effort);

$w_2 = E(y_2 \mid y_1) = E(\eta \mid y_1) = (1-\tau)E\eta + \tau(y_1 - \bar{a}_1) = \tau(y_1 - \bar{a}_1)$. The bigger the uncertainty, the more important y_1.

2.3.8 A continuum of types

Suppose now that θ is distributed according to the density $f(\theta)$ (with CDF $F(\theta)$) on $[\underline{\theta}, \bar{\theta}]$. Thanks to revelation principle, the seller's problem can be written as follows:

$$\underset{T(\theta),q(\theta)}{Max} \int_{\underline{\theta}}^{\bar{\theta}} [T(\theta) - cq(\theta)] f(\theta) d\theta$$

$s.t.$

(IR) $\theta u(q(\theta)) - T(\theta) \geq \underline{U} = 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$

(IC) $\theta u(q(\theta))-T(\theta)\geq\theta u(q(\hat{\theta}))-T(\hat{\theta})$ for all $\theta,\hat{\theta}\in[\underline{\theta},\overline{\theta}]$

[or $\theta\in\arg\max[\theta u(q(\hat{\theta}))-T(\hat{\theta})]$]

$\because\ \overline{\theta}u(q(\overline{\theta}))-T(\overline{\theta})\geq\overline{\theta}u(q(\underline{\theta}))-T(\underline{\theta})\geq\underline{\theta}u(q(\underline{\theta}))-T(\underline{\theta})=\underline{U}$, so we can replace the participation constraints by

(IR') $\underline{\theta}u(q(\underline{\theta}))-T(\underline{\theta})\geq 0$

(1) The implementation problem

At first, S-M condition is

$$\frac{\partial}{\partial\theta}[-\frac{\partial u/\partial q}{\partial u/\partial T}]>0$$

Secondly, we write FOC and SOC of IC conditions as follows:

FOC: $\theta u'(q(\hat{\theta}))\frac{dq(\hat{\theta})}{d\hat{\theta}}-T'(\hat{\theta})=0$. That is local incentive compatibility (LIC) conditions when $\theta=\hat{\theta}$.

SOC: $\theta u''(q(\hat{\theta}))(\frac{dq(\hat{\theta})}{d\hat{\theta}})^2+\theta u'(q(\hat{\theta}))\frac{d^2q(\hat{\theta})}{d\hat{\theta}^2}-T''(\hat{\theta})\leq 0$

Differentiating FOC with respect to θ, we obtain

$$\theta u''(q(\theta))(\frac{dq(\theta)}{d\theta})^2+u'(q(\theta))\frac{dq(\theta)}{d\theta}+\theta u'(q(\theta))\frac{d^2q(\theta)}{d\theta^2}-T''(\theta)=0$$

Together with SOC, it implies that $u'(q(\theta))\frac{dq(\theta)}{d\theta}\geq 0$. Since $u'(q(\theta))>0$, we get

$\frac{dq(\theta)}{d\theta}\geq 0$ (Monotonicity).

We have proved the necessity of monotonicity and LIC. Thirdly, we can prove the sufficiency of these properties by contradiction. We omit the process, see BD (2005, p84).

(2) The optimization problem

The seller's problem can therefore be written as

$$\underset{T(\theta),q(\theta)}{Max}\int_{\underline{\theta}}^{\overline{\theta}}[T(\theta)-cq(\theta)]f(\theta)d\theta$$

s.t.

(IR) $\underline{\theta}u(q(\underline{\theta}))-T(\underline{\theta})\geq 0$ (4-7)

$\theta u'(q(\theta))=T'(\theta)$ (4-8)

$$\frac{dq(\theta)}{d\theta} \geq 0 \tag{4-9}$$

Following Mirrlees (1971), define the indirect utility function

$$W(\theta) \equiv \theta u(q(\theta)) - T(\theta) = \max_{\hat{\theta}}[\theta u(q(\hat{\theta})) - T(\hat{\theta})]$$

By envelope theorem, we obtain

$\frac{dW(\theta)}{d\theta} = u(q(\theta))$, or integrating, $W(\theta) = \int_{\underline{\theta}}^{\theta} u(q(x))dx + W(\underline{\theta})$. According to IR conditions, we know $W(\underline{\theta}) = 0$, so the buyer's utility is

$$\theta u(q(\theta)) = T(\theta) + \int_{\underline{\theta}}^{\theta} u(q(x)) f(x) dx \tag{4-10}$$

By integration by parts, we also can get the seller's profits

$$V = \int_{\underline{\theta}}^{\overline{\theta}} \{[\theta u(q(\theta)) - cq(\theta)] f(\theta) - u(q(\theta))(1 - F(\theta))\} d\theta \tag{4-11}$$

Expression (4-11) differentiating with respect to $q(\theta)$, we get

$$[\theta - \frac{1 - F(\theta)}{f(\theta)}] u'(q(\theta)) = c \tag{4-12}$$

Let $h(\theta) \equiv \frac{f(\theta)}{1 - F(\theta)}$ (*hazard rate*), if it increase with θ, we can satisfy the monotonicity constraint.

Conclusions:

- The better the agent's type, the more he consume (Monotonicity);
- The better the agent's type, the bigger his utility (4-10);
- The better the agent's type, the more information rents he gets (4-10);
- There is underconsumption for all types $\theta < \overline{\theta}$, and when $\theta = \overline{\theta}$, $F(\theta) = 1$, $q(\theta^*) = q(\theta^{FB})$ (4-12).